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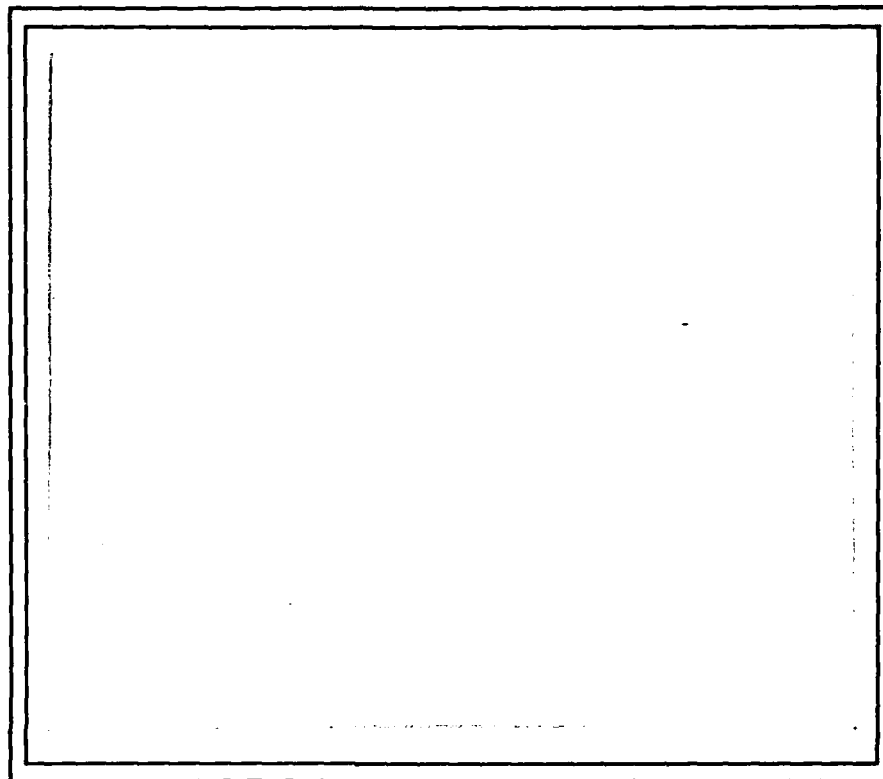
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9 Technical Rept. No. 54  
Civil Engng. Res. Rep. No. 80-SM-7

14 TR-54

6 NONLOCAL STRESS FIELD  
AT GRIFFITH CRACK.

by Cemal

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Research Sponsored by the  
Office of Naval Research  
15 under  
Contract N00014-76-C-0240 Mod 4  
Task No. NR 064-410

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# NONLOCAL STRESS FIELD AT GRIFFITH CRACK

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## ABSTRACT

By means of nonlocal elasticity theory, the Griffith crack problem is solved to determine the state of stress in a plate weakened by a line crack. The maximum stress is found to be finite and occurs away from the crack tip. Cohesive stress is estimated and compared with the calculations based on atomic lattice theory.

## 1. INTRODUCTION

In several previous papers, we have investigated the state of stress in plates containing sharp line cracks [1, 2, 3] and in Volterra dislocations [4,5]. These and our other similar work indicated that the nonlocal theory eliminates the stress singularity predicted by the classical elasticity theory. These results enabled us to introduce a natural fracture criterion based on maximum stress hypothesis and calculate the theoretical cohesive strength of various crystalline materials.

In reference [1], the maximum stress was predicted to occur at the crack tip while the solution of screw dislocation problems showed that the maximum stress occurs slightly away from the edge of the dislocation. According to a lattice model of crack used by Elliot [6], the maximum stress occurs slightly away from the crack tip. This situation was also confirmed

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in the recent computer simulations of Gohar [7]. To obtain such fine results near the crack tip, computer calculations must include discretization of a dimension less than one atomic distance and must also overcome the major difficulty related to the round-off errors. In [1], such fine resolution was not introduced and it prevented the prediction of the exact location of the maximum stress. Moreover, in [1], two-dimensional generalization of a one-dimensional nonlocal kernel was used.

In classical elasticity theory, a mixed-boundary value problem, prescribing the tractions on one part of the boundary surface and the displacement fields on the remainder, is well-posed. In nonlocal elasticity, this is not the case because of the fact that the prescription of the stress field at one point requires the knowledge of the strain field in the neighborhood of the point. Since at the crack tip this region extends into the body, it is necessary to verify the self-consistency of the boundary conditions in this overlapping region. Without such consideration, the boundary conditions within two atomic distances around the crack tip is satisfied in a non-uniform approximate manner (Atkinson [8]).

The *raison d'être* of the present work takes its origin from the foregoing considerations. In Section 2, we introduce a two-dimensional kernel by matching the dispersion curve of a square lattice to that of nonlocal plane waves. In Section 3, a set of self-consistent boundary conditions are rederived. Section 4 deals with the solution of the Griffith crack problem. Analytical expressions are given for the stress field along the crack line. The maximum stress and its location are calculated. In Section 5 we compare these results with those known in the atomic lattice theory and computer simulation studies. Quantitative agreements are gratifying.

## 2. TWO-DIMENSIONAL KERNEL

From [9], for a two-dimensional square lattice, we have the dispersion relations

$$(2.1) \quad \omega^2 = \beta_0 \{2 - \cos(\xi a) - \cos(\eta a) + 2 [1 - \cos(\xi a) \cos(\eta a)]\}$$

$$(2.2) \quad \omega^2 = \beta_0 [2 - \cos(\xi a) - \cos(\eta a)]$$

where  $\xi$  and  $\eta$  are Fourier variables corresponding to  $x$  and  $y$  directions,  $a$  is atomic lattice parameter and

$$(2.3) \quad \beta_0 = \frac{2}{3}(c_1/a)^2, \quad c_1 = [(\lambda + 2\mu)/\rho]^{1/2}$$

Here,  $c_1$  is the phase velocity of the irrotational waves,  $(\lambda, \mu)$  are Lamé constants and  $\rho$  is the mass density.

In nonlocal elasticity for isotropic solids, the stress constitutive equations are given by [10].

$$(2.4) \quad t_{k\ell} = \iint_R [\lambda'(|\underline{x}' - \underline{x}|) e_{rr}(\underline{x}') \delta_{k\ell} + 2\mu'(|\underline{x}' - \underline{x}|) e_{k\ell}(\underline{x}')] da(\underline{x}')$$

where the integral is over the two-dimensional plane region  $R(x', y')$  and  $e_{k\ell}$  is the strain tensor which is related to the displacement vector  $u_k$  by

$$(2.5) \quad e_{k\ell} = \frac{1}{2} (u_{k,\ell} + u_{\ell,k})$$

Here, and throughout an index following a comma represent gradient, e.g.

$$u_{k,l} = \partial u_k / \partial x_l$$

In this section we determine the kernels  $\lambda'$  and  $\mu'$  by comparing (2.1) and (2.2) with the dispersion of plane waves in nonlocal elasticity.

Substituting (2.4) and (2.5) into the equations of motion

$$(2.6) \quad t_{kl,k} - \rho \ddot{u}_l = 0$$

and taking the two-dimensional Fourier transform of the ensuing field equations, we obtain

$$(2.7) \quad [(\bar{\lambda}' + 2\bar{\mu}')\xi^2 + \bar{\mu}'\eta^2 - \rho\omega^2]\bar{u} + (\bar{\lambda}' + \bar{\mu}')\xi\eta\bar{v} = 0$$

$$(\bar{\lambda}' + \bar{\mu}')\xi\eta\bar{u} + [(\bar{\lambda}' + 2\bar{\mu}')\eta^2 + \bar{\mu}'\xi^2 - \rho\omega^2]\bar{v} = 0$$

where a superposed bar denotes Fourier transform

$$(2.8) \quad \bar{u}(\xi, \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) \exp[i(\xi x + \eta y)] dx dy$$

For a non-trivial solution of (2.7) to exist, the determinant of the coefficients of  $\bar{u}$  and  $\bar{v}$  in (2.7) must vanish, leading to dispersion relations.

$$(2.9) \quad \omega^2/c_1^2 = (\xi^2 + \eta^2)(\bar{\lambda}' + 2\bar{\mu}')/(\lambda + 2\mu)$$

$$(2.10) \quad \omega^2/c_2^2 = (\xi^2 + \eta^2)\bar{\mu}'/\mu$$

Comparing (2.9) with (2.1) and (2.10) with (2.2), we obtain expressions for  $\bar{\lambda}'$  and  $\bar{\mu}'$ .

Eventually, our main interest is in the nature of the singularity of  $\lambda'$  and  $\mu'$ . An examination  $\lambda'$  and  $\mu'$  so obtained showed that they both possess logarithmic singularities. In fact, inverse transform of  $\bar{\mu}'$  yields, for the case of Poisson material ( $\lambda = \mu$ ):

$$(2.11) \quad \mu'(x,y) = 2a^{-2} \ln \{ [(x-a)^2 + y^2][(x+a)^2 + y^2][x^2 + (y-a)^2][x^2 + (y+a)^2] (x^2 + y^2)^{-4} \}^{\frac{1}{4}}$$

It is clear that  $\mu'$  is not an isotropic kernel. However, noting that it has a logarithmic singularity, we may construct an isotropic kernel with the same singularity for both  $\lambda'$  and  $\mu'$ . The following kernel satisfies the basic requirements of a two-dimensional kernel.

$$(2.12) \quad \alpha(|\underline{x}' - \underline{x}|) = \frac{1}{2\pi} \beta^2 K_0 \{ \beta [ (x' - x)^2 + (y' - y)^2 ]^{\frac{1}{2}} \}$$

where  $K_0$  is the zeroth order modified Bessel function,  $\beta$  is a parameter, and

$$\alpha = \lambda'/\lambda = \mu'/\mu.$$

We note the following basic properties of  $\alpha$ :

- a) It has logarithmic singularity at the origin
- b) It vanishes as  $|\underline{x}' - \underline{x}| \rightarrow \infty$
- c) It's integral over the plane is unity, i.e.

$$(2.13) \quad \iint_R \alpha(|\underline{x}' - \underline{x}|) d\mathbf{a}(\mathbf{x}') = 1$$



- d) As  $\beta \rightarrow \infty$ , it reverts to a Dirac delta measure, so that (2.4) gives classical Hookes law.

Also, this kernel possesses an interesting property in that it satisfies the following differential equation

$$(2.14) \quad \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \beta^2 \right) K_0(\beta r) = \beta^2 \delta(r)$$

where  $\delta(r)$  is the Dirac delta function.

This property is very convenient for the reduction of the integro-differential field equations of the theory of nonlocal elasticity.

To determine  $\beta$ , we match the dispersion curve (2.2) at the end of Brillouin zone. This was done for two extreme cases.

$$(2.15) \quad 1/\beta = 0.22 a \quad (\xi = \pi/2a, \quad \eta = \pi/2a)$$

$$(2.16) \quad 1/\beta = 0.31 a \quad (\xi = \pi/2a, \quad \eta = 0)$$

The maximum error in these cases are less than 1% and 0.5%, respectively.

By integrating  $\alpha$  over  $y'$  from  $-\infty$  to  $\infty$  we can also derive a one-dimensional kernel which is of the form

$$(2.17) \quad \alpha(|x'-x|) = \frac{1}{2} \beta \exp[-\beta |x'-x|]$$

It must be noted that none of these kernels possess finite support, therefore special attention should be paid to the asymptotic behavior of the solutions involving Fourier integrals.

### 3. BOUNDARY CONDITIONS

A plate, in the  $(x,y)$  plane, containing a line crack of length  $2\ell$  along the axis of the rectangular coordinates, is subjected to a uniform tension  $t_0$  at  $y \equiv \infty$ , in the  $y$ -direction (Figure 1). This problem, which constitutes the foundation of contemporary fracture mechanics, is known as the Griffith crack problem, first treated by Griffith [11]. In the classical elasticity treatment, appropriate boundary conditions are:

$$\begin{aligned}
 (3.1) \quad & t_{xy}^c = 0, \quad y = 0, \quad \forall x \\
 & t_{yy}^c = -t_0, \quad y = 0, \quad |x| < \ell \\
 & v = 0, \quad y = 0, \quad |x| \geq \ell
 \end{aligned}$$

where  $t_{kl}^c$  represents the classical stress field. The Griffith crack problem requires the addition of a uniform stress field  $t_{yy}^c = t_0$  to the solution obtained under the boundary conditions (3.1). These boundary conditions do not carry over to the nonlocal elasticity in a strict sense. This is because at the crack tips  $(x = \pm \ell, y = 0)$  both the traction and the displacement fields cannot be prescribed simultaneously. Because of the integral form (2.4) of the stress constitutive equations prescription of the stress field at the crack tip requires that strain field must be known in a region around the tips. Due to the short range of interatomic force, this range is of the order of few atomic distances. Nevertheless, for proper continuum formulation, it is necessary to state the boundary conditions in a self-consistent form. To this end we observe that

- (a) In the limit as  $\beta \rightarrow \infty$  the nonlocal stress field must approach to the local stress field.
- (b) The stress field must be continuous in  $\beta$  for all  $x$  and  $y$  and possess continuous partial derivatives with respect to  $\beta$  for all  $\beta$ .

The stress constitutive equations (2.4) of the nonlocal theory may be expressed as

$$(3.2) \quad t_{kl} = \iint_R \alpha(|x' - x|) \sigma_{kl}(x') da(x')$$

where  $\sigma_{kl}$  is the local stress tensor given by the Hookes Law:

$$\sigma_{kl} = \lambda e_{rr} \delta_{kl} + 2\mu e_{kl}$$

If we express  $\sigma_{kl}$  in the asymptotic form

$$(3.3) \quad \underline{\sigma}(x, \beta) = \underline{f}_0(x) + \sum_{n=1} \underline{f}_n(x) \beta^{-n}.$$

Using (3.3) in (3.2) and applying the nonlocal smoothness conditions

$$(3.4) \quad \lim_{\beta \rightarrow \infty} \beta^{1+n} \frac{\partial^n}{\partial \beta^n} \{ \beta^{n-1} \underline{t}(x, \beta) \} = 0; \quad n > 0$$

we obtain restrictions on the functions  $\underline{f}_n(x)$ .

For the Griffith's problem, this leads to

$$(3.5) \quad \left. \begin{aligned} f_{0yy}(x) &= -t_0 \\ f_{0xy}(x) &= 0 \end{aligned} \right\} \quad |x| < l$$

and all other  $f_n(x)$  vanish inside of the crack line.

Boundary conditions of the nonlocal theory, appropriate to the Griffith problem, are therefore given by

$$\begin{aligned}
 (3.6) \quad & t_{xy}^c(x,0) = \sigma_{xy}(x,0) = 0, & y=0, \quad \forall x \\
 & t_{yy}^c(x,0) = \sigma_{yy}(x,0) = -t_0, & y=0, \quad |x| < l \\
 & v = 0 & y=0, \quad |x| \geq l
 \end{aligned}$$

Note that these boundary conditions are identical to those of the classical theory (3.1). This is due to the fact that the applied tension  $t_0$ , along the crack, is constant. For a non-uniform loading,  $t_0$  in (3.6) is replaced by a more complicated function and the functions  $f_n$  may not vanish. As a result, boundary conditions will be complicated by the requirement that  $t_{xy}^c = 0$  is no more valid along the entire x-axis.

#### 4. THE SOLUTION

Following Eringen, et al [1], for the field equations of the two-dimensional nonlocal elasticity, we have

$$\begin{aligned}
 (4.1) \quad & \iint_R \alpha(|\underline{x}' - \underline{x}|) [(\lambda + \mu)(u'_{,x'x'} + v'_{,x'y'}) + \mu \nabla'^2 u'] dx' dy' \\
 & - 2\mu \int_{-l}^l \alpha(|\underline{x}' - \underline{x}|) [e'_{yx}(x', 0)] dx' = 0 \\
 & \iint_R \alpha(|\underline{x}' - \underline{x}|) [(\lambda + \mu)(u'_{,x'y'} + v'_{,y'y'}) + \mu \nabla'^2 v'] dx' dy' = 0
 \end{aligned}$$

where

$$(4.2) \quad [e_{yx}(x', 0)] = e_{yx}(x', 0^+) - e_{yx}(x', 0^-)$$

For the kernel we use (2.12). Applying the operator in (2.14), using the Fourier transform along the x-direction

$$(4.3) \quad \tilde{f}(k, y) = (2\pi)^{\frac{1}{2}} \int_{-\infty}^{\infty} f(x, y) \exp(ikx) dx$$

we obtain the general solution of (4.1)

$$(4.4) \quad \begin{aligned} u &= (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \frac{1}{k} \left[ |k| A(k) + \left( |k|y - \frac{\lambda+3\mu}{\lambda+\mu} \right) B(k) \right] \exp(-|k|y - ikx) dk \\ v &= (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \left[ A(k) + yB(k) \right] \exp(-|k|y - ikx) dk \end{aligned}$$

The boundary condition (3.6) leads to a pair of dual integral equations for  $A(k)$  whose solution determines  $A(k)$  and  $B(k)$ :

$$(4.5) \quad \begin{aligned} A(k) &= A_0 J_1(\kappa) / \kappa \\ B(k) &= B_0 |k| A(k) \end{aligned}$$

where

$$(4.6) \quad \begin{aligned} A_0 &= (2/\pi)^{1/2} [2\mu (\lambda+\mu)/\ell^2 t_0(\lambda+2\mu)] \\ B_0 &= (\lambda+\mu)/(\lambda+2\mu) \\ \kappa &= k\ell \end{aligned}$$

Upon substituting (4.5) into (4.4) and (3.2), we obtain displacement and stress fields. Relevant to the present work is the hoop stress along the crack line, which is given by:

$$(4.7) \quad t_{yy}(\xi, 0)/t_0 = - \int_0^{\infty} \alpha(\epsilon \kappa) J_1(\kappa) \cos(\kappa \xi) d\kappa$$

where

$$(4.8) \quad \xi = x/l, \quad \epsilon = 1/\beta l$$

$$(4.9) \quad \alpha(\epsilon \kappa) = [(1 + \epsilon^2 \kappa^2)^{1/2} + \epsilon \kappa]^{-1} + \epsilon \kappa [(1 + \epsilon^2 \kappa^2)^{1/2} + \epsilon \kappa]^{-2}$$

$\alpha(\epsilon \kappa)$  given by (4.9) is much simpler than (5.11) obtained in [1].

The integral in (4.7) was evaluated by use of the asymptotic expansion of  $J_1(\kappa)$ :

$$(4.10) \quad J_1(\kappa) \sim \kappa^{-1/2} \cos(\kappa - 3\pi/4) + O(\kappa^{-3/2}); \quad |\kappa| \rightarrow \infty$$

The result to the order one ( $\epsilon^0$ ), is

$$(4.11) \quad t_{yy}(\xi, 0)/t_0 = (\pi/2)^{1/2} \left\{ \frac{\lambda^{1/2}}{\epsilon} [I_{1/4} K_{3/4} + I_{3/4} K_{1/4}] + \frac{\lambda^{1/2}}{2\epsilon} [I_{1/4} K_{3/4} - I_{5/4} K_{7/4} + I_{3/4} K_{1/4} - I_{7/4} K_{5/4}] \right\}$$

where

$$(4.12) \quad \lambda = \xi - 1$$

and  $I_\nu, K_\nu$  are  $\nu^{\text{th}}$  order modified Bessel functions and they are evaluated at  $\lambda/2\epsilon$ .

At the crack tip  $\xi = 1$  and we obtain

$$(4.13) \quad t_{yy}(1,0)/t_0 \simeq 0.57446 \epsilon^{-1/2}$$

We notice however, that the hoop stress is not maximum at the crack tip. In Table 1 we list the hoop stress in the vicinity of the crack tip as a function of  $\lambda/2\epsilon$ . From this table it is clear that

$$(4.14) \quad t_{yy \text{ max}} = 1.12 t_{yy}(1,0)$$

and this occurs at

$$(4.15) \quad \xi = 1 + 0.50 \epsilon$$

Using the values of  $\epsilon$  given (2.15) and (2.16), we obtain

$$(4.16) \quad t_{yy \text{ max}}/t_0 = 0.81 (2l/a)^{1/2}, \quad \xi \simeq 1 + 0.1 a/l$$

$$(4.17) \quad t_{yy \text{ max}}/t_0 = 0.97 (2l/a)^{1/2}, \quad \xi \simeq 1 + 0.15 a/l$$

## 5. COMPARISON WITH ATOMIC LATTICE THEORIES

The hoop stress along the crack line, given by (4.11), is plotted in Figure 2 against  $\xi = x/l$  for  $\epsilon = 0.22 a/l$ . On this figure, the hoop stress obtained by Elliot [6], and Eringen et al. [1] are also shown. The maximum stress and its location and the ratio of the maximum stress to

the stress at the crack tip are compared in Table 2. In Elliot's calculations,  $2l/a = 1000$  and the Poisson's ratio  $\nu = 0.25$ . We have therefore used the same numbers in (4.16) and (4.17) and the quantitative agreement is very good.

Gohar's [7] recent computer simulations on the crack problem in a two-dimensional triangular lattice indicates that the hoop stress profiles are of the same character as those of reference [1] and the present. He found that  $t_{\max}/t_0$  is generally larger than nonlocal continuum results while his stress profiles were somewhat broader. Strict comparison with his results may not be meaningful because of the inherent nonlinearity in the force potentials used in his model. However, in some cases, the maximum stress and the stress profile were found to be in good quantitative agreement with the nonlocal model.

Cohesive stress can be calculated by use of (4.16) or (4.17) in the same manner as in [1 - 5]. If one uses (4.16), one obtains 10% higher cohesive stress and if (4.17) is used, the cohesive stress becomes 30% higher than those given in reference [1].

The nonlocal fracture criteria introduced by Eringen [12] can be used in the same manner by setting  $t_{\max} = t_{\text{cohesive}}$ .

The present work modifies the boundary conditions used previously in [1] to eliminate overdetermination arising from the overlapping influence region of the stress and displacement fields on the boundary. Introduction of a new two-dimensional kernel brings the nonlocal model closer to two-dimensional lattice models. Nevertheless, numerical results are very close to those in [1], probably because the influence of the boundary "overlapping region" is extremely small, not affecting the outcome appreciably. The present approach enables us to state that the maximum stress does not occur at the crack tip.



TABLE 1  
HOOP STRESS NEAR THE CRACK TIP

$\lambda/2\epsilon$	$t_{yy}(\xi,0)/t_{yy}(1,0)$
0.0	1.
0.1	1.09
0.2	1.10
0.25	1.12
0.3	1.10
1.0	0.89
1.5	0.75
2.0	0.66

TABLE 2  
HOOP STRESS ALONG THE CRACK LINE

	$t_{\max}/t_0$	$\epsilon_{\max}$	$t_{\max}/t_{\text{tip}}$
Elliot [6]	27.62	$1 + 0.2 (a/l)$	1.16
Nonlocal Ref. [1]	23.08	--	--
Nonlocal } $1/\beta a = 0.31$	25.61	$1 + 0.15 (a/l)$	1.12
Present } $1/\beta a = 0.22$	30.67	$1 + 0.1 (a/l)$	1.12

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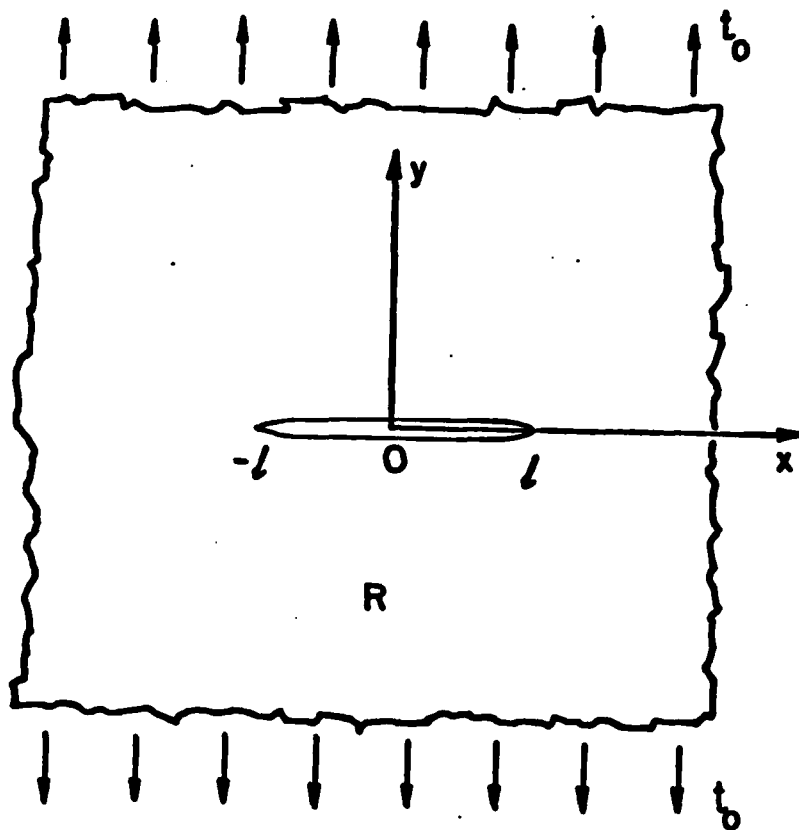
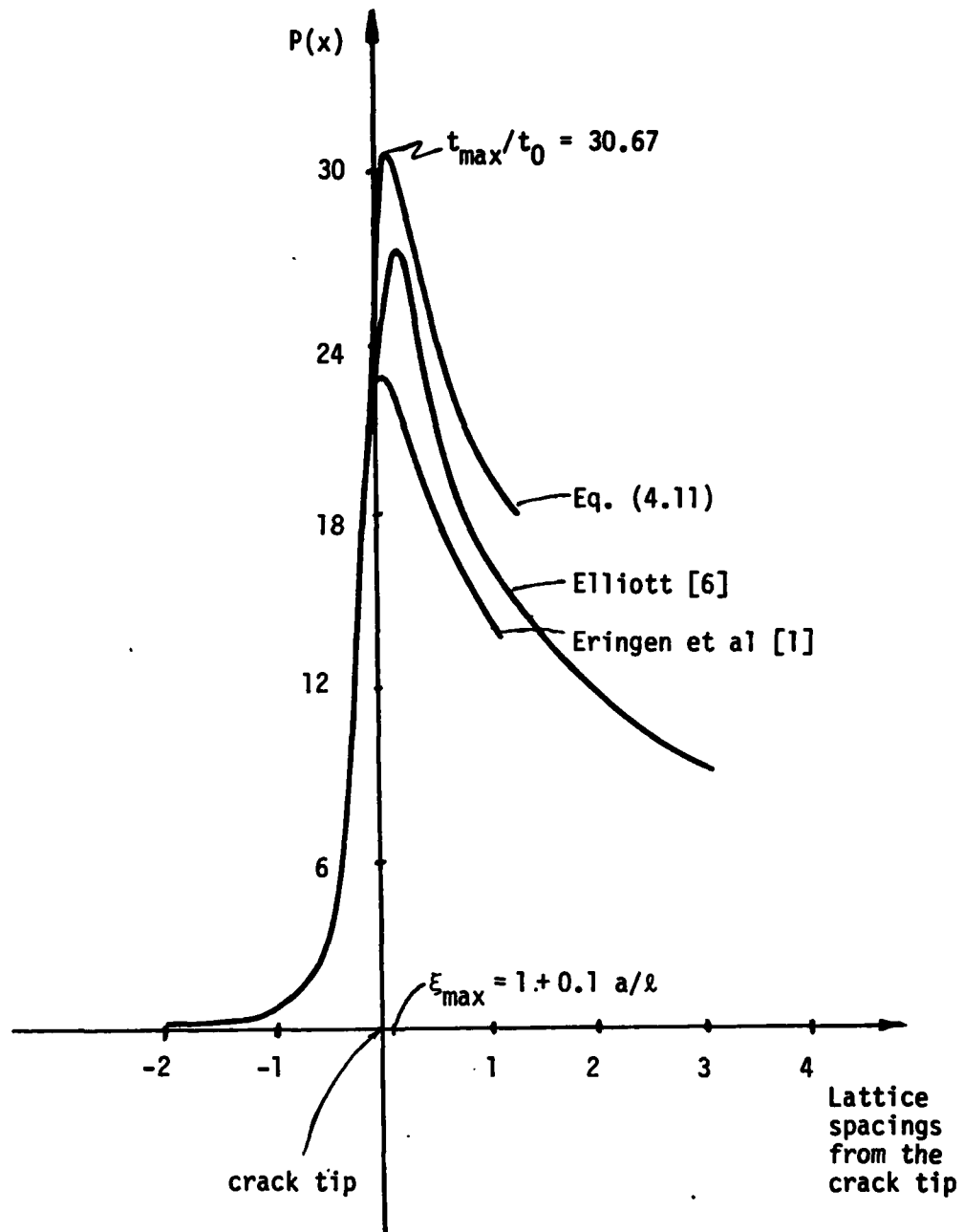


FIGURE 1

PLATE WITH LINE CRACK



$$P(x) = t_{yy}(\xi, 0)/t_0 + 1$$

FIGURE 2  
HOOP STRESS NEAR THE CRACK TIP

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1. REPORT NUMBER TECHNICAL PRINCETON UNIVERSITY REPORT #54	2. GOVT ACCESSION NO. AD-A086496	3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) NONLOCAL STRESS FIELD AT GRIFFITH CRACK		5. TYPE OF REPORT & PERIOD COVERED Technical Report	
		6. PERFORMING ORG. REPORT NUMBER 80-SM-7	
7. AUTHOR(s) N. Ari and A. Cemal Eringen		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0240 Mod. 4	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Princeton University Princeton, New Jersey		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 064-410	
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research (Code 471) Arlington, VA 22217		12. REPORT DATE June 1980	
		13. NUMBER OF PAGES 17	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) crack tip problem nonlocal elasticity theoretical cohesive stress fracture criterion			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) By means of nonlocal elasticity theory, the Griffith crack problem is solved to determine the state of stress in a plate weakened by a line crack. The maximum stress is found to be finite and occurs away from the crack tip. Cohesive stress is estimated and compared with the calculations based on atomic lattice theory.			

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U.S. Army Materials and Mechanics  
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U.S. Army Missile Research and  
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Air Force Office of Scientific Research  
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Attn: Mechanics Division

Department of the Air Force  
Air University Library  
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Director Defense Research  
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